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SIMULATION OF STEADY-STATE FLOW IN THREE-DIMENSIONAL FRACTURE NETWORKS USING THE BOUNDARY ELEMENT METHOD

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INTRODUCTION

The modelling of transport phenomena in fractured rock has been a topic of increasing interest over the past several years. In studies which have been undertaken to date, the means by which transport phenomena are mathematically conceptualized have taken two distinct routes. The necessity for alternative idealizations of fractured rock has arisen from the fact that the length scale of a given problem in relation to the density of fracturing is not consistent from one rock formation to another. This has led to conceptualizations of fractured rock as either a system of individual and possibly interconnected fractures in a permeable or impermeable host rock, or as one or more overlapping continua, in a similar manner to the mathematical treatment of granular porous media.

In applying the continuum conceptualization to a fractured rock, it is assumed that a sufficient number of interconnected fractures exist to define statistically meaningful properties associated with the fracture network at all points in the investigated domain. The physical structure of the individual fractures is no longer considered in the modelling procedure. Instead, average characteristics and responses are hypothesized for both the rock and the fluid phases within the fractures. It is only in this context that rock properties such as porosity and conductivity have meaning.

Clearly, the continuum conceptualization is applicable to those formations where there is a high degree of fracturing in relation to the problem scale. For rock formations where there are few fractures, it is necessary to consider the processes of transport in the network of individual fractures. Therefore, the explicit geometry of the fracture network plays an important role in the responses which are predicted. In this paper we concern ourselves with the discrete fracture conceptualization, and address the problem of modelling steady-state fluid movement in the fractures of an impervious host rock.

In modelling subsurface hydrologic phenomena, reducing the spatial dimensionality of a given problem is desirable for computational reasons. However, when considering discrete fracture networks, it is necessary to retain the three-dimensional geometric characteristics. Only in the case of specific fracture configurations, subject to specific fluid responses, can the three-dimensional problem of flow in a fractured rock be reduced to an equivalent two-dimensional problem. Such fracture geometries, however, are not often physically realistic.

Although it is necessary to consider the three-dimensional nature of a fracture network, it is possible to attain certain efficiencies in the solution procedure by making assumptions with regard to the fluid movement within the individual fractures. The fractures may be of any orientation within the rock, however, we assume flow in the plane of the fracture to be essentially two-dimensional. Such an assumption is appropriate if the fracture aperture is small in relation to its overall extent. The lines along which fractures intersect are also considered as (one-dimensional) fluid conduits. Under these assumptions it is possible to construct the three-dimensional geometric characteristics of flow in a fractured formation by considering a series of coupled, one- and two-dimensional equations.

GOVERNING EQUATIONS

Fractures

The movement of fluid at any point in a fracture, under the assumptions of steady-state flow and constant mass density, is described by

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{V} = 0 \tag{1}$$

where $\underset{\sim}{V}$ is the fluid velocity. Since an impervious host rock is being considered, we impose the condition of no flow at the fracture walls, i.e.,

$$\underset{\sim}{V} \cdot \underset{\sim}{n} = 0 \tag{2}$$

where $\underset{\sim}{n}$ is a unit normal vector outwardly directed from the fracture.

The fractures are assumed to be sufficiently narrow in relation to their overall extent. Therefore, flow is essentially two-dimensional in the surface defining the fracture axis. An equation of fluid movement which is appropriate under such conditions is generated by integrating (1) over the thickness of the fracture in the direction orthogonal to the fracture axis (Shapiro and Andersson, 1983). With the use of (2), the resulting two-dimensional equation takes the form

$$\underset{\sim}{\nabla}' \cdot (b\underset{\sim}{V}') = 0 \tag{3}$$

where $\underset{\sim}{V}'$ is the average fluid velocity defined over the fracture thickness, b, and $\underset{\sim}{\nabla}'$ is the gradient operator in the orthogonal, two-dimensional coordinate system which defines the fracture axis.

The product $b\underset{\sim}{V}'$ represents the fluid discharge over the thickness of the fracture at a given point. We shall assume that the fluid discharge is proportional to the gradient of an average hydraulic potential, h, defined over the fracture thickness, i.e.,

$$b\underset{\sim}{V}' = -K\,\underset{\sim}{\nabla}'h \tag{4}$$

where the fracture conductance, $K\ (L^2/T)$, is assumed to be a scalar quantity.

Substituting (4) into (3) yields

$$\underset{\sim}{\nabla}' \cdot (K\,\underset{\sim}{\nabla}'h) = 0 \tag{5}$$

This expression is valid at all points in the two-dimensional surface defining the extent of a given fracture. The areal boundaries of a fracture are taken to be its abrupt ending in the impervious host rock, or at the lines of intersection with other fractures.

Fracture intersections

Lines of fracture intersections are treated as separate fluid conduits since the conductance along the axis of the intersection need not be the same as that of the fractures which form it. We shall assume that flow in the intersection is essentially in the direction of its axis. However, at points along the intersection fluid can enter from (or exit into) the adjacent fractures. From (1), which is valid at all points in the fracture intersection, an appropriate one-dimensional equation of fluid movement can be generated in a manner similar to the development of (3). The form of this equation is

$$\frac{d}{d\xi}(aV_\xi) - \sum_{\ell=1}^{N} q^\ell = 0 \tag{6}$$

where V_ξ is the average fluid velocity in the direction of the intersection axis (defined by the coordinate ξ), $a(\xi)$ is the cross-sectional area of the fracture intersection and q^{ℓ} denotes sources of fluid from the (N) fractures which form the intersection.

Assuming that the fluid discharge through the fracture intersection can be defined by

$$aV_{\xi} = -\kappa \ (dh/d\xi) \tag{7}$$

where κ (L^3/T) is the conductance of the intersection, (6) takes the form

$$\frac{d}{d\xi}(\kappa \frac{dh}{d\xi}) + \sum_{\ell=1}^{N} q^{\ell} = 0 \tag{8}$$

In this analysis, we shall assume that the hydraulic head at a given point in the fracture intersection is equal to the hydraulic head at the boundary of the fractures which form it. In addition, we shall limit our discussion by considering only planar fractures. Consequently, fracture intersections are linear.

BOUNDARY ELEMENT FORMULATION

The three-dimensional geometry of the fracture network is retained in the solution of (5) and (8). These equations, however, cannot be solved independently since the fluid entering (or being discharged from) the fracture intersection depends on the responses in the adjacent fractures. The responses in the fractures, in turn, cannot be evaluated without the fluxes from the fracture intersections acting as boundary conditions. These equations are linear, however, and may be coupled to obtain a simultaneous evaluation of both flow regimes.

In practice, most any numerical technique for solving equations of the type given by (5) and (8) can be employed in this analysis. Here, however, we choose to apply the boundary element method in order to avoid internally discretizing the areal domain of a given fracture. The boundary element method computes the steady-state fluid responses by considering the fluid potential and fluid flux at the boundaries of the fracture planes. Since the flux at the boundary of the fracture plane appears as a solution variable, further numerical manipulation in unnecessary in order to obtain the flux as an input to (8).

Fractures

A boundary element solution to (5) is developed by first multiplying this expression by a function ψ and integrating over the investigated domain, in this case the area of a given fracture plane, A, i.e.,

$$\int_{(A)} \psi \ \nabla^2 h \ da = 0 \tag{9}$$

Here we have assume the conductance of the fracture to be constant over the fracture plane. In addition, we have dropped the superscript associated with the gradient operator since it is understood that we are considering a two-dimensional domain. Successive use of the divergence theorem in the above expression yields

$$\int_{(A)} h\,\nabla^2\psi\,da + \int_{(S)} \left(\psi\,\frac{\partial h}{\partial n} - h\,\frac{\partial \psi}{\partial n}\right) ds = 0 \qquad (10)$$

where S is the closed boundary defining the areal domain and $\partial(\ )/\partial n$ is the outwardly directed normal derivative to the boundary S.

The function ψ is chosen to be a solution to $\nabla^2\psi + \delta(x-x_m)\,\delta(y-y_m) = 0$ where $\delta(\ )$ is the Dirac delta function. In two-dimensions, ψ is the logarithmic potential function, i.e.,

$$\psi(x,y;x_m,y_m) = -\frac{1}{2\pi}\,\ell n(\{(x-x_m)^2+(y-y_m)^2\}^{\frac{1}{2}}) \qquad (11)$$

With the definition of ψ, (8) can be written for an arbitrary point (x_m,y_m) on the boundary as (Brebbia, 1978)

$$\frac{1}{2}\,h(x_m,y_m) = -\int_{(S)} \left(\psi\,\frac{q}{K} + h\,\frac{\partial \psi}{\partial n}\right) ds \qquad (12)$$

where in place of $\partial h/\partial n$ we have introduced the outwardly directed fluid flux vector, $q=(b\underset{\sim}{V}'\cdot\underset{\sim}{n})$, on the boundary of the fracture plane.

Approximate solutions for h and q on the boundary are obtained by discretizing the previous equation. As an assumption, over each boundary element on S, h and q are taken to be constant. Consequently, the discretized form of (12) is

$$\frac{1}{2}\,h_m = -\sum_{j=1}^{E} \left\{ \frac{q_j}{K} \int_{\Delta S_j} \psi_{jm}\,ds + h_j \int_{\Delta S_j} \frac{\partial \psi}{\partial n}_{jm}\,ds \right\} \qquad (13)$$

where E is the number of elements comprising S with each element denoted by ΔS_j, h_j and q_j are the element values of the hydraulic head and fluid flux, respectively, and the subscripts j and m denote nodal locations on the boundary.

At boundary elements, other than those at the fracture intersections, either h or q is prescribed as a boundary condition. At those elements which lie along a fracture intersection, both h and q are unknown, and must be solved in conjunction with a discretized form of (8).

Fracture intersections

Along fracture intersections, the discretization is chosen to be consistent with that of the fracture planes. At the midpoint of each element we write the following discretized form of (8),

$$\frac{\kappa}{(\Delta\xi)^2}\,(h_{j-1} - 2h_j + h_{j+1}) + \sum_{\ell=1}^{N} q_j^{\ell} = 0 \qquad (14)$$

where the subscript j denotes the element under consideration and $\Delta\xi$ is the length of the boundary elements (which are assumed to be constant) along a given fracture intersection.

At points where fracture intersections intersect, we apply a condition of continuity with regard to the fluid fluxes along the intersection axes, i.e.,

$$\sum_{i=1}^{I}(aV_{\xi})_i = -\sum_{i=1}^{I}(\kappa\,\frac{dh}{d\xi})_i = 0 \qquad (15)$$

where I denotes the number of lines of intersection at the point in question. At the end points of fracture intersections, it is possible to prescribe the hydraulic head (e.g., if the fracture intersection ends at a boundary of the rock domain) or a fluid flux (e.g., if the intersection ends at a point internal to the rock domain, the flux is zero).

A determinant system of equations is provided by writing (13) for all fracture planes, and (14), (15) and the appropriate boundary conditions for all fracture intersections. The solution variables are the hydraulic head and the fluid flux at the boundaries of the fracture planes.

<u>Treatment of wells</u>

In our analysis, we additionally consider the possibility of boreholes from which fluid can be extracted. For those fractures which are intersected by a borehole, the borehole is treated as an internal boundary. The shape of the borehole in the fracture plane is discretized by a series of boundary elements. This is entirely consistent with the form of (13) where E can be considered as the total number of boundary elements, including those elements which define the boreholes in the fracture plane.

Since a given borehole may intersect several fractures, the fluid production from each of the fractures is only a portion of the total discharge. Furthermore, the amount of fluid discharged from each fracture, and the distribution with respect to the elements which define the borehole is unknown. Over the length of the borehole we shall assume that the hydraulic head is constant. Although this acts as a boundary condition applied to the elements defining the borehole, the value of the hydraulic head which allows the correct discharge from all fractures is unknown. Therefore, for boreholes which intersect more than one fracture plane, it becomes necessary to iteratively obtain a solution, i.e., to prescribe a value of the head in the borehole and check if the solution for the total discharge from all fractures satisfies the prescribed discharge from the well.

NUMERICAL RESULTS

The accuracy of the boundary element method as applied to problems similar to this is demonstrated in Shapiro and Andersson (1983). Here, we consider a hypothetical example of steady-state fluid movement in a three-dimensional fracture network in order to demonstrate the boundary element formulation presented above. We consider the three-dimensional section of a fractured rock with the dimensions shown in Fig. 1. In this domain there are three planar fractures (F1, F2 and F3) forming two fracture intersections. In addition, we consider the presence of a borehole of radius 0.1 m, which is vertically oriented, and located at a position which intersects the three fractures. Fracture planes F1 and F2 are assigned a conductance of 0.75 m^2/day, while the conductance of F3 is given a value of 10 m^2/day. The conductance of the fracture intersections are each assigned a value of 20 m^3/day. At the rock faces in the x-direction ($x=0$ and $x=-40$), we impose the condition of a prescribed hydraulic head, $h=0$. For all other rock faces an impervious boundary is assumed.

For these physical conditions, and for the parameter values given, we examine the fluid responses in the fracture network as a result of injecting fluid in the borehole at a rate of 100 m^3/day. Results for this sample problem are given in Fig. 2 where lines of equipotential are plotted for those segments of the fractures which are intersected by the borehole. The values of the hydraulic head at the interior points of the fracture planes are obtained from the solutions for h and q on the boundaries (see, Brebbia, 1978).

The results indicate large hydraulic gradients in the vicinity of the borehole in fractures F1 and F2. This arises due to the lower conductance assigned to these fractures. Although there is a larger hydraulic gradient in the vicinity of the borehole in F1 and F2, the percent of the total injected fluid which enters these fractures is only 11.5 and 10.5 percent, respectively. The majority of the injected fluid enters the fracture network through F3.

SUMMARY AND CONCLUSIONS

An efficient method of predicting steady-state fluid responses in three-dimensional fracture networks is formulated with the use of the boundary element method. The three-dimensional geometric characteristics of the fracture geomentry are retained while solving coupled sets of one- and two-dimensional equations. In this analysis, fractures may have any orientation in an impervious host rock. However, the fractures are treated as surfaces where fluid movement is assumed to be essentially two-dimensional. In using the boundary element method, the steady-state fluid responses in the fractures are evaluated by considering the hydraulic head and fluid flux at the boundaries of the fracture planes. These boundaries are depicted as either the abrupt ending of the fracture in the impervious host rock, or as lines of intersection with other fractures.

Fracture intersections, which are also treated separate fluid conduits, are described by one-dimensional equations written in terms of the hydraulic head and the fluid flux from those fractures which form the intersection. These are the same solution variables which define the fluid responses in the fracture surfaces. Consequently, a direct coupling of the discretized equations for each flow regime is easily attained.

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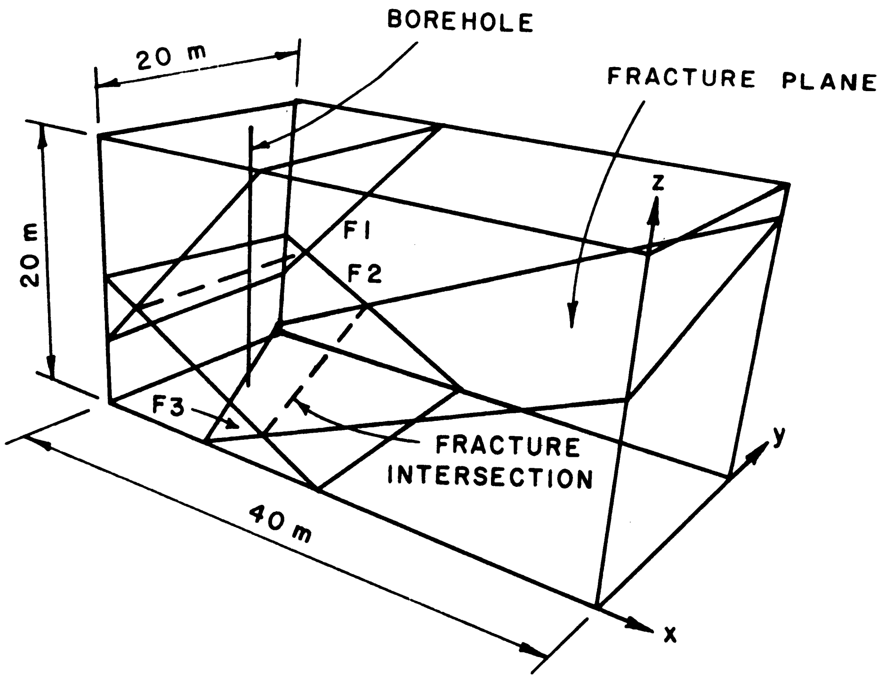

Figure 1 Three-dimensional fracture geometry of the sample problem.

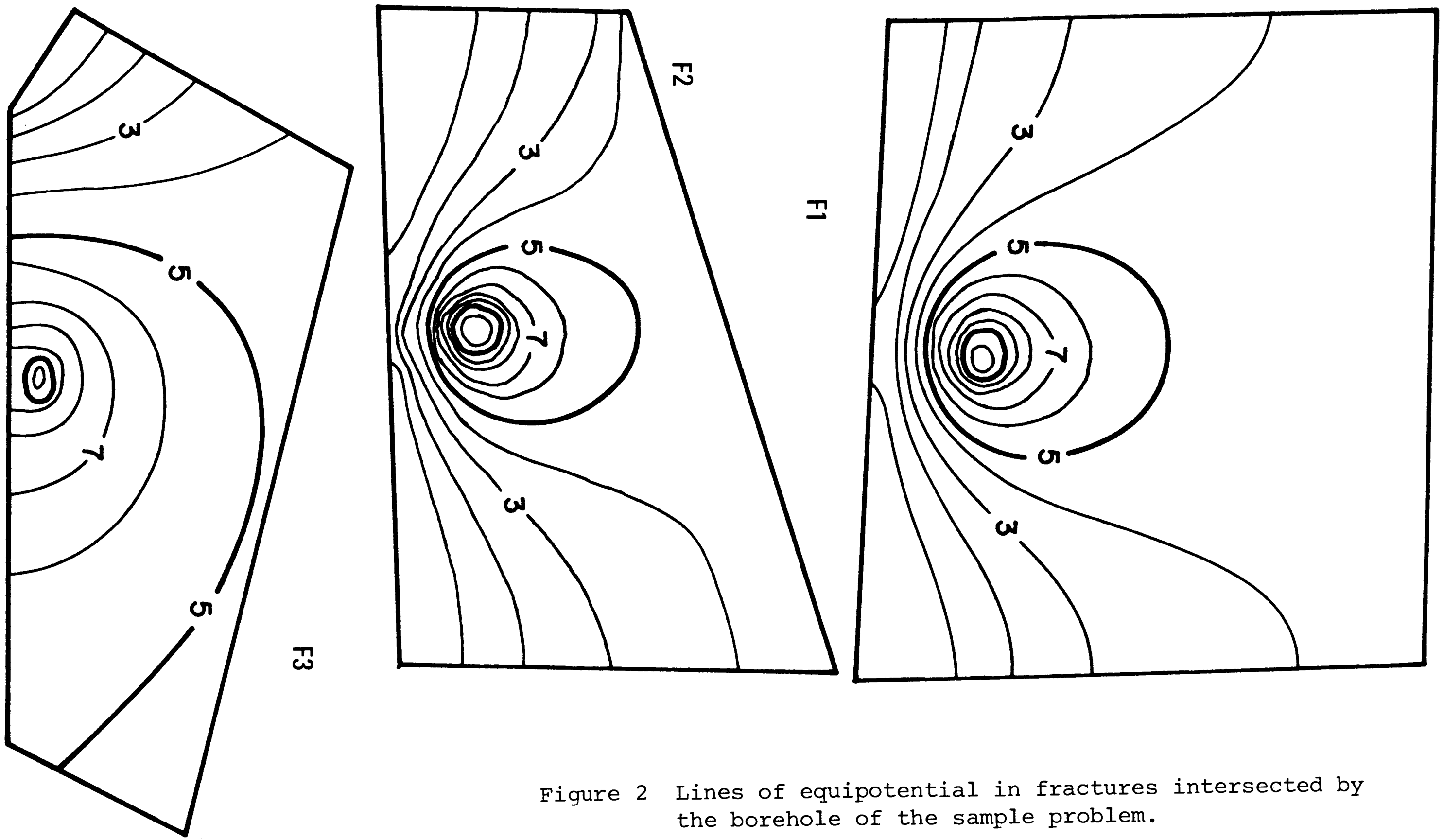

Figure 2 Lines of equipotential in fractures intersected by the borehole of the sample problem.